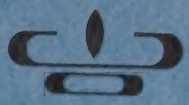

ONTARIO
DEPARTMENT OF
EDUCATION

General and Advanced Levels of Instruction in Grade 13 Mathematics

JANUARY, 1966

As recommended by the General and Advanced Committee which conducted part of the Grade 13 studies, the suggested outlines listed below have been prepared at the direction of the Minister of Education by the Curriculum Division with the assistance of university and secondary school personnel. The outlines are to be considered as working papers rather than definitive courses. It is expected that they would be subject to further development and change if it is decided to submit them to field testing. They are being distributed to the universities and to the secondary schools for evaluation.

GA. 31 Accountancy and Secretarial Studies

GA. 13 Art

GA. 17B Biology

GA. 11 Classics

GA. 18 Chemistry

GA. 4 English

GA. 46 français

GA. 15A French

GA. 17E General Science

GA. 7 Geography

GA. 15B German

GA. 9 History

GA. 12 Mathematics

GA. 16 Music

GA. 17C Physics

BROAD SUGGESTIVE OUTLINES OF PROPOSED GENERAL AND ADVANCED COURSES IN MATHEMATICS

The proposals of this committee are based on the agreement that the General Course will probably be a terminal course in mathematics for the majority of the students who take it, while the Advanced Course is to provide the basis for further mathematical study and application. In considering the courses in this light, it was also agreed that the General Course should provide mathematics which will be useful to the student. At the same time this course should include topics which develop mathematical maturity, are interesting and which help the student to see that mathematics is a growing, dynamic subject rather than a static body of facts. In the Advanced Course, the mathematics that will be studied and used by the student at University is the determining factor. These considerations led to the decision that not only the content but also the format of the courses should differ.

THE GENERAL COURSE IN MATHEMATICS

This course consists of a compulsory topic together with a variety of optional topics. The Compulsory topic is Probability and Statistics to which approximately 55% of the time (45 hours) has been allotted. There are six optional topics of which two would be chosen, each requiring approximately 18 hours.

The committee agreed that, of the possible choices for the compulsory topic, "Probability and Statistics" would be the most useful. The optional topics were chosen to provide an appreciation of some aspects of mathematics which are interesting and of increasing importance.

This "core-plus" organization allows for the introduction or deletion of optional topics. This produces a flexibility in the curriculum to meet future needs, an opportunity for experimentation, and the means whereby the curriculum can be changed easily without a major revision of the entire course.

The presentation of Probability and Statistics should be based on experiments and in a laboratory type of environment. The calculating machines referred to in the Introduction to the course can be purchased by the schools for $200 to $300 each. A calculating laboratory with from 10 to 15 such machines would normally suffice for the purposes of this course. It is expected that a calculating laboratory of this type would be used to advantage in other courses.

The committee recommends that no *external* examination be given in this course for two reasons:

1 the computational aspects of the course would make external testing by examination impracticable

2 an external examination for a General Course would tend to make it impossible for a teacher to adapt the topics of the course to the needs and interests of the students.

MATHEMATICS

Proposed Course for the General Level

INTRODUCTION

This course has been designed for the student who does not intend to pursue an extensive study of mathematics after Grade 13. The development of the course is based on two main objectives:

1 to give the student a working knowledge of Probability and Statistics

2 to give the student an appreciation of other topics in mathematics which have increasingly important applications.

The approach to the topics should be intuitive rather than rigorous. The presentation should be designed to arouse the student's interest and to encourage him to study in greater depth those topics which appeal to him most.

The course consists of the following units:

Unit I: Probability and Statistics
Unit II: Introduction to Calculus
Unit III: Algebraic Structures
Unit IV: Linear Programming
Unit V: Further Applications of Calculus
Unit VI: Vectors
Unit VII: Matrices and Markov Processes

The minimum requirement of the course is three units (eighty-one hours) to be as follows:

(a) Unit I: (45 hours)

(b) one of Units II and III (18 hours)

(c) one other unit not chosen in (a) or (b): (18 hours)
(Note: Unit II is a prerequisite for Unit V)

The possible combinations of the options are:

1 Units I, II, and III
2 Units I, II, and one of IV, V, VI, or VII
3 Units I, III, and one of IV, VI, or VII

In the ideal situation, the choice of options would be made by the student; until this is possible, teachers should attempt to assess the needs and interests of a class to assure that the choice of options is suitable.

The outline for each unit is suggestive only. Order of presentation, extent and depth of treatment should be varied to fit the individual situation.

Unit I — Probability and Statistics (45 hours)

The presentation of this topic should include an integration of probability and statistics. Sampling and selection procedures should arise throughout the work rather than appear as a separate topic.

This unit assumes the use of adding machines with multiplication facility.

The same data should be used as far as possible throughout to allow calculations done in previous sections to be used in subsequent developments.

It is recommended that this study be considered a laboratory subject and that students work in small teams as much as possible to reduce tedious calculations for each individual.

This topic includes:

1 Analysis of data: sigma notation, measures of central tendency, measures of dispersion and calculating methods

2 Probability: historical introduction, definition of probability, mutually exclusive events

3 *r*-Subsets and *r*-Arrangements: an introduction to *r*-subsets (combinations) and *r*-arrangements (permutations) and applications to probability

4 Frequency Distributions: binomial and normal distributions

5 Sampling: bias, random samples, sample means, standard error, hypothesis testing (including non-parametric methods)

Unit II—An Introduction to Calculus (18 hours)

Any attempt, at the Grade 13 level, to give a student an appreciation of the scope of mathematics, should include an elementary introduction to the calculus. This introduction relies on an intuitive development of the necessary concepts, and proceeds to simple applications of the derivative to mathematical and physical problems.

Topics included are:

1 Graphs and Slopes: historical introduction to calculus, graphs of functions, slopes of tangents

2 Derivatives: definition, notations, derivatives of simple functions

3 Applications: maxima and minima, curve tracing, distance, velocity and acceleration.

Unit III — Algebraic Structures (18 hours)

By the end of Grade 12, the student has become familiar with the structure of the Real Number System. This unit provides an introduction to a study of groups, using a variety of numerical and non-numerical examples. Operations on ordered pairs lead to an examination of the group properties of two dimensional vectors, complex numbers, and 2×2 matrices.

Unit IV — Linear Programming (18 hours)

This unit introduces an application of inequalities which is becoming increasingly important in ndustry.

The unit includes:

1 Graphical Methods: inequalities in the plane, convex sets in the plane, simple two variable linear programming problems, solved graphically

2 Systems of Linear Equations: Gauss-Jordan elimination and applications

3 The Simplex Method: slack variables, simplex method, problems solved by the simplex method.

Unit V — Further Applications of Calculus (18 hours)

This unit is an extension of Unit II and is for those students who find an interest in the calculus. The applications of the calculus are extended to include other types of problems and situations.

Topics included:

1 Rules for Differentiation: an extension of the rules developed in Unit II

2 Applications of Derivatives: an extension of the applications of Unit II with an emphasis on physical applications

3 Derivatives of Exponential Functions

4 Equations of the form $D_x y = f(x)$: solution by inspection, applications to geometry, kinematics, growth and decay.

Unit VI — Vectors (18 hours)

This topic presents an introduction to the algebra of vectors and its applications.

This unit includes:

1 Preliminary Solid Geometry: an intuitive introduction to the geometry of three dimensions

2 Solid Analytic Geometry: Cartesian co-ordinates in three dimensions, equations of lines and planes, distance and midpoint formulas

3 Vector Analysis: elementary vector algebra and applications.

Unit VII — Matrices and Markov Processes

This unit introduces the algebra of matrices and applications to problems arising in commerce. The second topic in the unit, Markov processes, applies probability and matrices to problems in the social sciences.

THE ADVANCED COURSE IN MATHEMATICS

The students taking the Advanced Course would, in general, be those proceeding to further study involving mathematics and the sciences. Such a course forms the basis for further studies and cannot have the flexibility of a General Course.

The content of the new courses in Grade 13 Mathematics — "Introduction to Analysis" and "Algebra" — can be developed into the Advanced Course.

As an Advanced Course, this material would be assigned six hours of instruction per week (in addition to private study time). Until the General — Advanced programme is introduced, nine hours (15 periods per week would continue to be allowed.)

This use of the new courses as the basis for the Advanced Course minimizes the problem of teacher preparation. However, the material in the two new courses should be reorganized for the best presentation as a single Advanced Course.

The topics included in the two new courses are presented here. For further details reference should be made to Curriculum S.12C.

INTRODUCTION TO ANALYSIS

Unit 1 — Logical Reasoning

This unit forms a short introduction to the general principles of proof (and disproof) in mathematics and to the vocabulary of logic.

Unit 2 — Function as a Mapping

The study of function began in Grade 12 is pursued here with an emphasis on its interpretation as a mapping of one set into another.

Unit 3 — Second Degree Relations in the Plane

This unit extends the Grade 12 study of the second degree relations to develop the simpler properties of the conics.

Unit 4 — Trigonometry

The Grade 12 course provides a study of the sine and cosine functions. This unit extends the study to the other four related functions and provides a development of the sum and difference formulas.

Unit 5 — Transformation in the Plane

In this unit, translations, rotations, and reflections in the plane are studied as mappings. The transformations are then used to simplify second degree equations in order to identify the particular conics.

Unit 6 — Slopes and Simple Derivatives

This unit begins an intuitive approach to calculus placing emphasis on the graphical and physical interpretation of the derivative, which in this treatment is defined as a new function f' such that $f'(x)$ equals the slope of the graph of the function at the point $(x, f(x))$.

Unit 7 — Applications of Differentiation

Many of the applications appearing in this unit will have been encountered by the pupils in previous experiences in mathematics and science. It is the objective of this unit to give practice with these practical problems.

Unit 8 — Equations of the Form $Dxy = f(x)$

This unit introduces applications of the calculus to problems in analytic geometry, kinematics and areas.

ALGEBRA

Unit 1 — Sets, Subsets, and Permutations

This unit introduces some algebra of sets, basic counting principles, r-arrangements (permutations) and r-subsets (combinations).

Unit 2 — Mathematical Induction and the Binomial Theorem

Unit 3 — Probability

This unit is designed to provide an introduction to the topic of probability. The development of the topic is restricted to situations that yield only a finite number of possible outcomes. Solutions of problems may be by the use of tree diagrams or by the use of sample spaces.

Unit 4 — Vectors

The first sections of this unit are designed to provide an introduction to vectors without the use of a coordinate system. In each of the sections of this introduction it is and examples for motivation and for clarification of the intended that there will be frequent use of illustrations concepts involved. Sections 8, 9, 10, 11, 12, 13, introduce coordinate systems for the study of vectors. Sections 14, 15, and 16, introduce the measurements associated with vectors.

Unit 5 — Equations of Lines and Planes

This unit extends the work on vectors in Unit 4 to its uses in 2 and 3 dimensional geometry.

Unit 6 — Systems of Linear Equations

This unit extends the work of earlier grades to the solution of m equations in n unknowns by matrix methods.

Unit 7 — Matrices and Linear Transformations

In this unit, matrices and the application of matrices to linear transformations are introduced. While the definitions are general, practice with the operations and applications is to be restricted to cases where the number of rows and columns is two.

Unit 8 — Complex Numbers and Polar Co-ordinates

Unit 9 — Examples of Groups

This unit introduces the definition of groups, rings and fields and uses examples from previous parts of the course, e.g., transformations, number systems, functions, and matrices.

APPENDIX A — MATHEMATICS

Proposed Course for the General Level

Unit I — Probability and Statistics (45 hours)

The following outline for this Unit is to indicate the material that should be covered, rather than an order of presentation. The presentation of this topic should include an integration of the probability and statistics. Sampling and selection procedures should arise throughout the work, rather than appear as a separate topic.

This unit assumes the use of adding machines with multiplication facility.

The same data should be used as far as possible throughout to allow calculations done in previous sections to be used in subsequent developments.

It is recommended that this study be considered a laboratory subject, that students work in small teams as much as possible to keep tedious calculation to a minimum.

1 ANALYSIS OF DATA (9 hours)

Sigma notation: examples to illustrate the use and properties of sigma notation for summation.

Organization of data, frequency diagrams.

Measures of central tendency: mean, median, mode.

Measures of dispersion — standard deviation, range, quartiles, mean deviation.

Calculation Methods — use of adding machines with multiplication ability, use of tables of squares, Bolzano's method for square roots.

2 PROBABILITY (9 hours)

Historical introduction

Tabulation of results of experiments.

Definition of the probability of an event where the number of outcomes is finite.

Mutually exclusive events: $p=p_1+p_2+p_3+ \ldots +p_k$

(Supplementary) Conditional probability: $p=p_1 p_2$

3 *r*-SUBSETS AND *r*-ARRANGEMENTS (6 hours)

The topic of *r*-arrangements is to be used only to develop the number of *r*-subsets of a set of *n* elements. A minimum of time should be used for arrangement problems. The work on *r*-subsets is to be directed towards probability applications.

Definition of an *r*-arrangement (permutation) of *n* things as an ordered selection or arrangement of *r* of them.

The study of particular cases to establish that the number of *r*-arrangements of *n* things is:

$n(n\text{-}1)\ (n\text{-}2) \ldots (n-r+1)$

Definition of an *r*-subset (combination) of a set of *n* elements as a selection of *r* of them without regard to order.

Development of the number of *r*-subsets of a set of *n* elements.

The notation $\binom{n}{r}$

Applications to probability

4 FREQUENCY DISTRIBUTIONS (4 hours)

Development of the expansion for $(p+q)^n$, *n* an integer; applications to probability.

Mean and standard deviation of binomial distribution.

Normal distribution as an approximation to the binomial distribution; graph from tables of ordinates; significance of areas under the normal curve; areas from tables; applications.

5 SAMPLING, HYPOTHESIS TESTING FOR LARGE SAMPLES (12 hours)

Bias, random samples, tables of random numbers. Sample mean, significance of the difference between the means of two samples.

Standard error

Hypothesis testing (including non-parametric methods)

Unit II — An Introduction to Calculus (18 hours)

1 GRAPHS (2½ hours)

Historical introduction to calculus

Sketching graphs of equations for the following:

straight line, $y=mx+b$

parabola, e.g., $y=x^2$, $y=2x^2+3x$, $y=2x^2+x+1$

cubics, e.g., $y=x^3$, $y=x^3-9$

hyperbolas, e.g., $y=\frac{1}{x}$, $y=\frac{2}{x+3}$

This study should lead to the use of the slope of the tangent at a point as a way of finding the position of hills and valleys of the graph.

2 SLOPES (6½ hours)

A review from Grade 12 of the concept of a tangent as the limit of secants.

Study of the graph of $y=x^2$ to determine the slopes of secants leading to the use of the concept of tangent as the limit of a sequence of secants.

The determination of the slope of tangents at specified points on the graph of $y=x^2$.

Similar study of the function defined by $y=x^3$.

Similar treatment at specified points on the graphs of equations, such as $y=2x^2+3x$, $y=x^3-9x$.

Determination of the slope of the graph at the point (x_1, y_1) for several functions, e.g., the functions defined by

$y=x^2$, $y=x^3$, $y=\frac{1}{x}$, $y=8x^3$, $y=6$, $y=2x^2+5x+3$

3 DERIVATIVES (2½ hours)

Definition of the derivative f of a function f.

The notations $D_x y$, $\frac{dy}{dx}$, $f'(x)$ for the value of the derivative at x.

Determination of a formula for the derivative, with respect to x, of the function defined by $y=x^n$, n, a non-negative integer.

Determination of the derivative, of a constant function, the sum and difference of a function.

4 APPLICATIONS OF DERIVATIVES (6½ hours)

Applications of the derivatives selected from maxima and minima; curve tracing; distance, velocity, acceleration.

Unit III — Algebraic Structure (18 hours)

1 BINARY OPERATIONS ON A SET

Physical, geometric, and arithmetic examples of binary operations.

Special types of binary operations (with examples), i.e. associative, commutative, idempotent, operation with identity, operations with inverses.

Uniqueness properties of an identity and of inverses.

2 GROUPS

Definition of a Group

—commutative and non-commutative groups

Examples: similitudes of rigid geometrical objects
permutations of two and three objects
arithmetic groups
modular arithmetic groups
the discovery that modular multiplication is a group operation only for a prime base.

Subgroups: examples from above
discovery of divisibility rule

3 VECTORS

Vectors as n-tuples, for $n=2, 3, 4$.

Vector addition as a commutative group operation.

Geometric interpretation from analytic geometry for $n=2$

4 COMPLEX NUMBERS

Definition, with addition, as per vectors with $n=2$

Definition of multiplication

Demonstration that the system of complex numbers is a field.

Discovery that $(0,1)^2=(-1,0)$.

One-to-one correspondence between the field of complex numbers of the form $(a, 0)$ and the field of real numbers (the idea of isomorphism might be introduced).

Notation: $(a,0) \rightarrow a$
$(0,1) \rightarrow i$
$(0,b)=(b,0)(0,1) \rightarrow bi$
$a+bi=(a,0)+(b,0)(0,1)=(a,b)$
$i^2=-1$

Rules of complex arithmetic using notation $a+bi$, alternate method of defining complex numbers.

5 MATRICES: (2×2 only)

Definition of a matrix and of matrix addition (essentially the same as for vectors with $n=4$)

Additive group of 2×2 matrices

Matric form for 2×2 linear equations (this in-

volves defining multiplication of matrices and vectors).

Substitution in linear equations as motivation for definition of matrix multiplication.

Identity for multiplication. Discovery of rule for a multiplicative inverse to exist.

Matrix representation of complex numbers.

Unit IV — Linear Programming

1 GRAPHICAL METHODS (6 hours)

Single inequalities in the plane.

Several inequalities in the plane.

Convex sets in the plane.

The value of c in $ax+by=c$ and its relation to the maximum value of $ax+by$ if (x, y) belongs to a convex polygon in the plane.

Statement of the theorem for a maximum of a linear form.

Simple two variable linear programming problems solved graphically.

Semigraphical solution by finding vertices of polygons algebraically and using trial and error.

Alternative requirements and solutions.

2 SYSTEMS OF LINEAR EQUATIONS (6 hours)

Simultaneous equations 2×2, 3×3 and 4×4 solved by Gauss-Jordan elimination, the use of detached coefficients and pivots; the solution in parametric form or recognition of inconsistency.

Solution of m equations in n unknowns $m < -n$ for $n=3$, 4 or 5 by Gauss-Jordan elimination.

Examination of solutions if inequality restrictions are placed on the variables.

3 THE SIMPLEX METHOD (6 hours)

Introduction of "slack" variables to permit replacement of some inequalities by equalities.

Development of the simplex method by example.

Two variable problems solved by the simplex method, checked by graphs.

Three variable problems solved by simplex method.

Unit V — Further Applications of Calculus (18 hours)

This unit is dependent on Unit II and would be chosen only if Unit II has been covered. Topic I of this unit could be taught before topic 4 of Unit II.

1 RULES FOR DIFFERENTIATION (3 hours)

Determination of the derivative with respect to x of a reciprocal $\frac{1}{u}$, a product uv, and u^n where u and v are differentiable functions of x.

Determination of the derivative of a quotient $\frac{u}{v}=u\times\frac{1}{v}$ using the reciprocal and product rules.

Practice in the calculation of derivatives involving sums, products, quotients and functions defined implicitly.

2 APPLICATIONS OF DERIVATIVES (6 hours)

An extension of the applications suggested in Unit II, topic 4.

Problems involving related areas.

3 DERIVATIVES OF EXPONENTIAL FUNCTIONS (3 hours)

Review of graph $y=10^x$

Graphical determination of slopes of tangents to $y=10^x$ leading to the conclusion that $D_x y=b(10^x)$ where $b \doteq 2.303$

Demonstration that if $y=10^{ax}$ then $D_x y=ab(10^{ax})$ determination of a such that $D_x(10^{ax})=10^{ax}$

4 EQUATIONS OF THE FORM $D_x y=f(x)$ (6 hours)

Solution by inspection of equations of the form $D_x y=f(x)$.

Finding the equation of a family of curves with a given slope.

Selection of a particular member of the family that satisfies a prescribed condition.

Finding the velocity-time relation given the acceleration relation.

Finding the distance-time relation given the velocity relation or the acceleration relation.

Other problems involving distance, velocity, acceleration and time.

Problems involving exponential growth and decay.

Unit VI — Vectors (18 hours)

1 PRELIMINARY SOLID GEOMETRY (3 hours)

This topic is intended to give the student an intuitive introduction to the geometry of three dimensions.

Intersections of lines and planes, planes and planes.

Perpendicularity and parallelism.

2 SOLID ANALYTIC GEOMETRY (3 hours)

Cartesian co-ordinate system in three dimensions.

Equations of lines and planes.

Distance and mid-point formulas

3 VECTOR ANALYSIS (12 hours)

Concept and definition of a vector.

Equality of vectors.

Length of a vector.

Unit basis vectors, $\mathbf{i}=(1, 0, 0)$, $\mathbf{j}=(0, 1, 0)$, $\mathbf{k}=(0, 0, 1)$

Scalar multiple of a vector.

Geometric interpretation and applications.

Linear dependence (optional).

Scalar Product: definition, applications.

Unit VII — Matrices and Markov Processes (18 hours)

It is assumed that all students who wish to take this option will have had the coverage of probability outlined in the core topic Probability and Statistics.

If the student has studied the work on matrices and vectors in the Algebraic Structures option then some of the work on Vectors and Matrices can be curtailed. Similarly, if the Linear programming option is taken the work on systems of linear equations can be omitted.

1 VECTORS AND MATRICES (6 hours)

Definition of row and column vectors and their rules of addition and multiplication.

Applications to business problems.

Definition of a matrix as an array and multiplication by a vector.

Applications to business problems.

Addition and multiplication of matrices.

Solution of systems of linear equations $(n \times n)$ by Gauss-Jordan (echelon) methods.

2 MARKOV PROCESSES (12 hours)

Stochastic processes — independent processes, independent trial processes and Markov processes.

The transition diagram and probability matrix, P, for a single step in a Markov processes.

Two and three step processes by direct computation.

Demonstration that $p^{(n)}=p^{(n-1)}$ and so $p^{(n)}=p^{(0)}P^n$, simple problems.

Intuitive ideas that pr tends to a unique limiting matrix as $n \to \infty$ and that the rows in this matrix are identical.

Discovery of a row in the limiting matrix as a vector such that $tP^n=t$ and statement of theorem for "regular" probability matrices.

Applications of regular Markov chains.

Applications to communications networks.

APPENDIX B

References

The Advanced Course in Mathematics

Since the Advanced Course is based on the present new courses for Grade 13, the text books for these courses provide the necessary reference and problems source.

The General Course in Mathematics

The General Course in Mathematics is a new course, independent of any particular text books or materials. The following list of books and pamphlets provides a variety of sources for the content of this course. None of these references covers the material for which it is a reference in exactly the same way that the course outline suggests. Any experimentation with this course will require the compilation of notes and problems.

The order of the references in the following list is not to be considered as an indication of the relative importance of the books.

A Some Lessons on Mathematics
Association of Teachers of Mathematics
Cambridge University Press

B, C Statistics (2 volumes)
Loveday
Cambridge University Press

D Introduction to Probability and Statistics
Alder and Roessler
W. H. Freeman and Co.

E Modern Introductory Analysis
Dolciani et al
Houghton Mifflin

F Introduction to Finite Mathematics
Kemeny, Snell and Thompson
Prentice-Hall

G Fundamentals for Advanced Mathematics
Glicksman and Ruderman
Holt, Rinehart, Winston

H Vector Analytic Geometry
Paul A. White
Dickenson

I Linear Programming and Theory of Games
Glicksman
Wiley

J What is Mathematics?
Courant and Robbins
Oxford

K Principles of Mathematics
Allendoerfer and Oakley
McGraw-Hill

L Introduction to the Theory of Groups
Alexandroff
Blackie

M Algebra and Vector Geometry
Stanton and Fryer
Holt Rinehart and Winston

N Topics in Modern Mathematics
Stanton and Fryer
Prentice-Hall

P Fundamentals of Freshman Mathematics
Allendoerfer and Oakley
McGraw-Hill

Q Basic Topics in Mathematics
Riner
Prentice-Hall

R Elementary Vector Geometry
Schuster
Wiley

S Modular Arithmetic
Jones
Blaisdell

T Vectors in Three Dimensional Geometry
Glicksman
National Council of Teachers of Mathematics

V A New Calculus
Seddon, Snell, Morgan
Cambridge University Press

Other useful reference books:

Linear Algebra; Crouch, Beckman; Scott Forsman

Enrichment Topics for High School; National Council of Teachers of Mathematics

The Mathematics of Matrices; Davis; Blaisdell

Contemporary School Mathematics Series
First Series: Matrices 1
Second Series: Matrices 2

An Introduction to Probability and Statistics; Macmillan

Elementary Matrix Algebra; Hohn; Macmillan

An Introduction to Linear Programming; Bristol; Heath

A Survey of Modern Algebra; Birkhoff and MacLean; Macmillan

What is Calculus About? W. W. Sawyer; Random House

Limits; Miller; Blaisdell

Algebra and Trigonometry, a Modern Approach; Peters and Scheff; Van Nostrand.

Page References

Unit I Statistics and Probability

B: 1-43
C: 3-9, 12-35, 61-81
D: 1-122
E: Ch. 15
F: 92-107, 113-133
G: 247-258, 262-293, 296-311
K: Ch. 13
M: 36-41
N: 145-163

Unit II An Introduction to Calculus

E: Ch. 8
G: Ch. 22, Ch. 23
J: 414-421, 423-425
N: 128-144
P: Ch. 16
V: Chs. 2, 3, 4

Unit III Algebraic Structures

A: 43-65
E: Ch. 7
G: Ch. 15, 325-335
J: 31-36, 88-97
K: 61-64, 69-82
L: 1-7, 15-25
M: 94-102, 214-215, 226-228, 295-307
N: 1-15, 65-77
P: Ch. 7
Q: Ch. 4
S

Unit IV Linear Programming

A: 208-222
F: 249-265
I: to p. 78
M: 306-307
Q: Ch. 5

Unit V Further Applications of Calculus

E: 363-369
P: Ch. 16

Unit VI Vectors

A: 300-317
G: 398-406, 482-501
H: 33-85
M: 215-253
R: 1-134
T

Unit VII Matrices and Markov Processes

E: 547-555
F: 139-156, 171-177, 178-212, 215-232, Ch. VII
N: 138-155